



Liquidity Constraints and the Centralized Home Mortgage Policy in China

Gao, Cixiu

Publication date:
2017

Citation for published version (APA):
Gao, C. (2017). *Liquidity Constraints and the Centralized Home Mortgage Policy in China.*

Liquidity Constraints and the Centralized Home Mortgage Policy in China

Cixiu Gao¹ and Aiyong Zhu²

¹Department of Economics, University of Copenhagen

²Department of Economics, Wuhan University

February, 2017

Abstract

This paper investigates China's centralized mortgage policy in general, and evaluates the policy change in 2011: the minimum down payment was dramatically raised to 60% of the home price. Using our previously developed structural model, we recover the unobserved liquidity wealth levels of individual borrowers. We provide quantitative evidence for the doubtful efficacy of the 2011 raise in the minimum down payment: it hurts the moderately-wealthy individuals, leaving them to struggle with high default risks, but doing little to prevent the rich from entering the market.

Keywords: Home mortgage, Minimum down payment; Liquidity constraints, China

JEL Classification Numbers: C54, D82, G21

1 Introduction

China's housing mortgage loan market is highly influenced by centralized government policy, which regulates nearly-uniform loan interest rate, maximum term, and minimum down payment, in addition to the usual terms and clauses in the mortgage contract.¹ In general, mortgage down payment serves as an insurance for the bank: it makes it more likely to recover the balance in the event of default. It also serves as a screening device to overcome the adverse selection problem, for instance in the subprime car loan market (Einav et al., 2012). In China, the minimum down payment, which normally ranges from 20% to 50%, seems to be serving as a macroeconomic regulatory device rather than a risk-control tool. For instance, in January 2011 when the market was overheated the ratio was raised to 60% for second-time home buyers,² aiming to calm it down; in the beginning of 2016 when apartment inventory was excessively high it was lowered to 20% in order to boost real estate demand. However, the well-being of individual borrowers and the lender's long-run risks are of less concern.

This paper investigates China's centralized mortgage policy in general, and evaluates the policy change in 2011. In the previous work, we developed a two-part model that describes a forward-looking risk-averse agent making dynamic default decisions and the optimal down payment choice, in an environment with uncertainties but no refinance. The structural estimates allow simulations of future default probabilities given the observed borrower characteristics. These simulations require borrower liquid wealth level as initial conditions for the dynamic system. Our previous model shows that, though this wealth level is unobserved by the econometrician, it can be (partially) recovered by exploiting the actual down payment, which is observed.

The model suggests that, regardless of the level of the uniform down payment requirement, it screens out the least wealthy borrowers, forces moderately-wealthy individuals to pay down more than they would have if they were to freely choose the down payment (the threshold borrowers), while leaving the wealthiest unconstrained by the minimum requirement (the non-threshold borrowers). It also suggests that at a higher minimum requirement, a poor borrower will quit the market; a moderately wealthy borrower stay in the market while being forced to pay down a larger amount and facing higher default risks; a sufficiently wealthy borrowers will be unaffected. Therefore, knowing the individual wealth levels is crucial not only for examining the borrower wealth composition and predicting default risks under actual down payment requirements, but also for simulating outcomes under counterfactual down payment requirements.

We then use the model to examine the impact of the 2011 raise. It is found that the new 60% minimum effectively screened out the low-wealth buyers, which to some extent may have helped prevent potential bubbles. However, the recovered wealth endowments suggest that an influx of unprecedentedly rich borrowers, unconstrained by the 60% minimum, emerged after 2011. The actual efficacy of the raise in cooling down the market is doubtful, because the uniform down payment requirement can do nothing to prevent the rich from entering the market.

A counterfactual experiment can examine the impact of a raise in more detail. Such examination would be impossible to perform using non-structural analysis because borrowers who would have paid down strictly less than 60% are not observed. The only subjects suitable for this experiment are the *pre-2011* second-time

¹The aims of the Chinese housing mortgage policies are believed to be macroeconomic, such as to maintain the value of the real estate market which is the main debtor of state banks, to boost the prosperity of the construction industry which is one of the major driving forces of GDP and employment, diminish real estate bubbles, etc. See Deng et al. (2009) for a literature review on Chinese housing reform and policy. Within the limits allowed by the centralized policy, banks do not seem to use risk-based pricing; this may be due to the lack of a credit score system.

²In this paper *first time buyers* refer to those who previously own no property, and *second time buyers* to those who own at least one. The two types of borrowers are treated differently by mortgage policies.

borrowers, as they are constrained by minimums *below* 60%.³ The result shows that at the higher minimum, about half of the observed pre-2011 second-time borrowers (1,074 out of 2,197) will leave the market, not being able to afford the larger down payment; among the 1,123 individuals who stay, 840 of them become the new threshold borrowers, choosing to pay down just 60%. The remaining 283 borrowers are not affected. The 840 new threshold borrowers with very even less liquid wealth remaining, will be extremely vulnerable: their average default risk is predicted to be as high as 38.13%.⁴

In the US, the Federal Housing Administration requires 3.5% down, substantially lower than the Chinese counterparts. The requirement on down payment is also lenient in European and other Asian countries. The second experiment aims to examine an extreme case, 0% down.⁵ The result shows that, without any down payment requirement, the subsequent well-being of the borrowers would be much greater than in the actual case: after down payment, the remaining wealth level is on average about 82% higher, and the long-run default rates are significantly lower, with a reduction of between 7.68% and 22.31%.

In sum, the paper shows how a raise in the minimum down payment drains the moderately-wealthy individuals, while leaving the rich intact. These findings remind the policy maker about the trade-off between the current real estate market stability, the borrowers' well-beings, and the lender's long-run risks. The purpose of the paper is not to recommend a "0% down". Instead, it highlights the negative policy effects of the heavy and uniform down payment requirement.

The paper is organized as follows. Section 2 describes the 2011 policy change in the minimum down payment. Section 3 outlines the policy implications of the model. Section 4 evaluates the 2011 raise policy, paying particular attention to liquid wealth. Section 5 examines alternative policies. Section 6 concludes.

2 Data and the 2011 Policy Change

The dataset is obtained from a regional commercial bank headquartered in a provincial capital of south-east China. The original panel contains individual information of all its 35,417 mortgage customers with mortgage contracts started from October 1998 to December 2014. It also includes the housing characteristics of the mortgaged property, contract terms, and the number of dates overdue. More details about the dataset can be found in the previous work.

Table 1 provides summary statistics. Column 1 displays a summary of all borrowers. Then the sample is divided up by borrower type; column 2 and 3 show respectively the first time and second time buyers. The second time buyers are further divided into buyers who originated the loan before and after January 2011, when two major dramatic changes were made: the minimum down payment was raised to 60% and the interest rate was raised to 10% above the base rate for all second time buyers. The two types seem to have similar preferences on loan terms. They are also similar in pre-owned versus new apartments; about half of the properties are previously owned.

Despite of these similarities, the two types are treated very differently by government policy, and the groups have quite different default risks. The first time borrowers have seemingly gotten better deals, obtaining interest rates that are on average 14.49% below the base rate, compared to the second time's

³The post-2011 borrowers are useless for this experiment, because those who would have participated had the minimum been below 60% are not observed. The experiment is a partial-equilibrium, in which property prices and all bank-chosen variables are assumed fixed, though we would expect property prices to fall at a harsher mortgage policy.

⁴The pre-2011 second-time borrowers' 38.13% is substantially higher than the prediction on the post-2011 second-time borrowers' average, 3.33% to 4.16%, only because the former group contains a large portion of newly emerged rich individuals whose default rates are predicted to be very low.

⁵This experiment is run on the first-time threshold buyers who are constrained by a 20% to 50% minimum (it is silent about those who did not participate under these minimums).

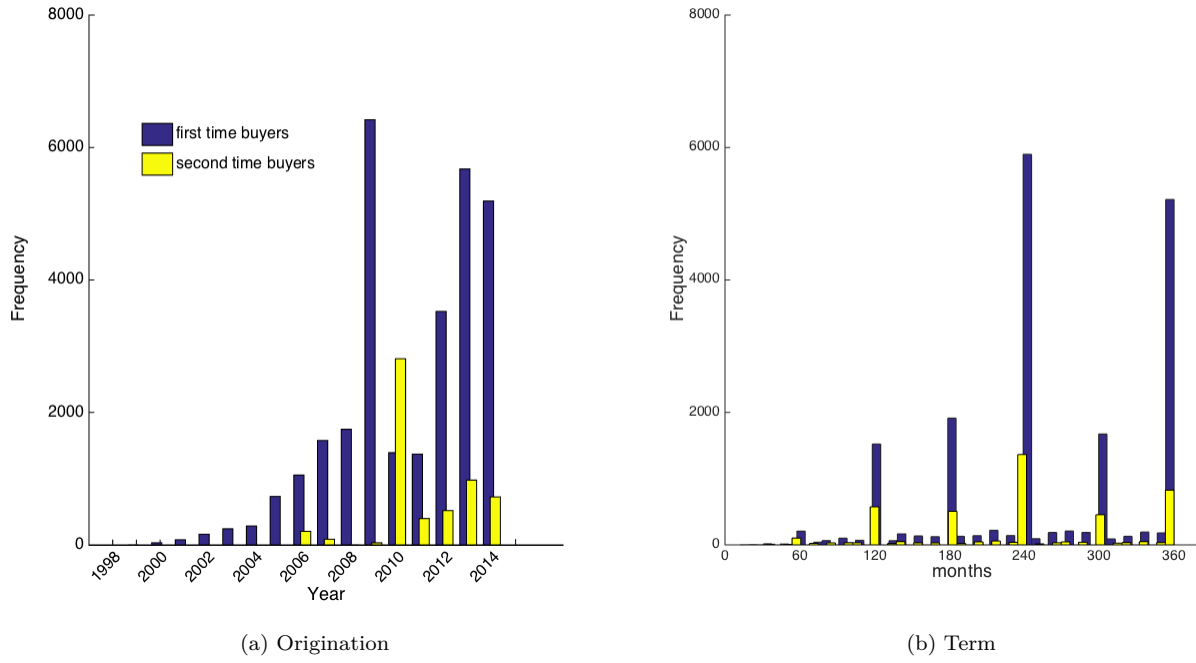


Figure 1: Choice Frequencies of Mortgage Term and Origination

3.93%. However, when dividing the second time borrowers into two groups, before and after January 2011, we see that the pre-2011 second times actually obtained slightly lower rates, 15.55% below the base rate; only after 2011, due to the new tough rules imposed on second time buyers, did they get interest rates 9.97% above the base rate (2294 out of 2618 got exactly 10%). One possible reason why the central bank have tightened loans for second time borrowers in 2011 can be seen from the rates of default: by December 2014, the pre-2011 second time borrowers have 1.33% chance of default, compared to only 0.28% of the first time borrowers. One reason why second time buyers are more likely to default is probably that they own another apartment they can live in, whereas such option is non-existent for first time buyers.

The 2011 policy is meant to reduce the number of potential second time borrowers and to prevent real estate bubbles. Figure 1a shows that the number of second time borrowers spiked in 2010 (perhaps a sign of a bubble, see also Cao et al. (2015)), though it slightly recovers after a great reduction in 2011. An interesting question is why the pre-2011 policies offered better deals to the riskier second time buyers. Another unresolved question is whether or not the tough 2011 policy can reduce the number of second time buyers and their long-run default risks.

3 Behavioral Model

The formal model consists two parts. Part 1 depicts the dynamic default choice making of a risk-averse forward-looking borrower conditional on some starting amount of liquid wealth, facing future uncertainties with no access to refinance. Part 2 describes the optimal choice of down payment subject to the minimum requirement. The dynamic default choices are subgame-perfect to the choice of down payment, conditional on the remaining wealth after the down payment. The formal part-1 model is presented in the previous

Table 1: Summary Statistics

	All		first time		second time		pre-2011 second time		post-2011 second time	
	mean	std.dev.	mean	std.dev.	mean	std.dev.	mean	std.dev.	mean	std.dev.
p_i	742.19	83.23	698.51	551.11	966.12	683.46	881.21	605.37	1067.74	754.17
$area_i$	98.73	40.66	97.79	39.45	103.56	46.06	98.32	43.38	109.82	48.35
$preowned_i$	0.52	0.50	0.51	0.50	0.57	0.50	0.55	0.50	0.58	0.49
no_i	0.16	0.39	0.00	0.00	1.03	0.17	1.05	0.23	1.00	0.00
M_i	244.67	81.92	246.21	81.23	236.77	84.98	247.76	84.32	223.62	83.91
$principal_i$	379.05	356.10	367.74	354.06	437.05	360.85	518.37	377.77	339.81	312.97
income	136.09	199.66	130.70	205.92	158.80	168.93	163.23	157.58	154.45	179.29
$dev(\%)$	-12.77	14.43	-14.49	13.31	-3.93	16.57	-15.55	14.15	9.97	2.98
Year	2010.69	2.74	2010.61	2.87	2011.07	1.88	2009.65	1.08	2012.78	1.02
# of inds.	23682		19140		4542		2250		2292	
# of defaults	85		53		32		30		2	
default risk	0.36%		0.28%		0.70%		1.33%		0.09%	
defaulter life	69.17	30.01	70.12	32.49	66.00	19.70	67.70	18.51	34.50	17.68

Principals, apartment values (p_i) and incomes are measured in 1,000 CNY of 1999 value. Mortgage term (M_i) and defaulter life are measured in months.

paper. This paper will focus on the policy implications. But for comprehensiveness we outline the optimal down payment choice making.

A borrower i is endowed with (liquid) wealth $w_{i,0}$. Before the game starts, she possesses the outside option: a first time borrower lives in some rental apartment, and a second time lives in the property she owns. She then takes into account future uncertainties and decides whether to purchase the property and take a mortgage loan, or to retain the outside option. If yes (opt in), she then chooses a down payment subject to the minimum down payment requirement, in order to maximize the expected total discounted utility; if not (opt out), she retains the value of the outside option, $V_{i,0}^{out}(w_{i,0}, \alpha_i)$.⁶ For the down payment choice, the trade-off faced by the borrower is between a bigger remaining wealth that can be used to deal with future shocks, and smaller monthly payment burdens.⁷

Let the down payment be denoted z_i . If the borrower pays down z_i , her remaining wealth is $w_{i,1}(z_i) = w_{i,0} - z_i$. She will borrow $p_i - z_i$ from the bank, and the per-period payment is $x_i(z_i) = (p_i - z_i)R_i$, where R_i is the “price of mortgage”, given by the standard formula of fixed-rate mortgage (FRM), and in period 1 the realized value of opt-in is $\bar{V}_{i,1}^{in}(w_{i,1}(z_i), x_i(z_i))$ (suppressing the state variable $t = 1$).⁸ The down payment choice is merely prior to all subsequent default choices and there involves no stochastic expense shocks ξ and there is no discounting between period 0 and 1. Also, in $t = 0$ there involves no choice of consumption. Borrower characteristics, including interest rate i_i , income y_i , term M_i , apartment value p_i and size h_i , first or second time buyer, are taken as given.

The borrower’s problem can be written as

$$\begin{aligned} \max & \left\{ \max_{z_i} \bar{V}_{i,1}^{in}(w_{i,0} - z_i, (p_i - z_i)R_i), V_{i,0}^{out}(w_{i,0}, \alpha_i) \right\} \\ \text{s.t. } & \eta_i p_i \leq z_i \leq p_i \end{aligned} \quad (1)$$

⁶The choice of apartment, which determines h_i and p_i , is not modeled in this paper. These decisions could be modeled to be made in a stage prior to down payment taking the bank-chosen mortgage variables as given, and down payment and default choices are subgame-perfect to apartment choice.

⁷Without uncertainties, the optimal down payment is a corner solution determined by the the saving and loan interest rates.

⁸This inside value is the so-called integrated value function in the DDC literature, the expected value of opt-in before knowing the idiosyncratic shock $\epsilon_{i,1}$: $\bar{V}_{i,1}^{in} = \int \max_d \{V_{i,1}^{in} + \epsilon_{i,1,0}, V_{i,1}^{out} + \epsilon_{i,1,1}\} dG(\epsilon_{i,1})$.

The constraint represents the minimum down payment requirement, in which η_i is minimum ratio requirement of borrower i . Let z_i^* be the optimal down payment *unconstrained* by the minimum down payment requirement conditional on opt-in. Suppose there exists a unique interior solution z_i^* . The first-order condition is

$$-\frac{\partial \bar{V}_{i,1}^{in}(w_{i,1}(z_i^*), x_i(z_i^*))}{\partial w_{i,1}} - R_i \frac{\partial \bar{V}_{i,1}^{in}(w_{i,1}(z_i^*), x_i(z_i^*))}{\partial x_i} = 0. \quad (2)$$

The interior solution z_i^* is a function of the unobserved wealth endowment $w_{i,0}$ and the observable borrower characteristics α_i :

$$z_i^* = z(w_{i,0}, \alpha_i). \quad (3)$$

The associated optimum is denoted $\bar{V}_i^{in}(z_i^*)$. It is straightforward that the borrower would still choose to opt in even if the interior solution z_i^* violates the minimum requirement constraint, as long as the inside value at the required minimum payment $p_i \eta_i$ is greater than the outside value. Formally, the optimal choice rule is as follows. If

$$z_i^* > p_i \eta_i \text{ and } \bar{V}_i^{in}(z_i^*) \geq V_{i,0}^{out}(w_{i,0}, \alpha_i) \quad (4)$$

then the agent becomes a non-threshold borrower with $down_i = z_i^*$. If

$$z_i^* \leq p_i \eta_i \text{ and } \bar{V}_i^{in}(p_i \eta_i) \geq V_{i,0}^{out}(w_{i,0}, \alpha_i) \quad (5)$$

then the agent becomes a non-threshold borrower with $down_i = p_i \eta_i$. If neither (4) nor (5) hold, then the agent chooses to opt out, obtaining $V_{i,0}^{out}(w_{i,0}, \alpha_i)$.

The model has a number of important implications.

1. **Income Effect** $0 < \frac{\partial z^*(w_0)}{\partial w_0} < 1$. This condition implies that if unconstrained by the minimum requirement, a wealthier borrower pay down a larger amount, and that she will have more wealth remaining after the down payment.
2. **Adverse Selection** Larger loans correlate with higher default risks. All else equal, those who choose to pay down weekly larger amount are those who have (1) weekly larger remaining wealth and (2) weekly smaller monthly payment burden, both of which implies smaller default risks.
3. **Policy Implication i** Given borrower characteristics α and a minimum down payment requirement η , there exists two cut-off wealth levels $\underline{w}_0(\eta, \alpha)$ and $\bar{w}_0(\eta, \alpha)$ satisfying $\underline{w}_0(\eta, \alpha) \leq \bar{w}_0(\eta, \alpha)$, such that a sufficiently poor individual ($0 < w < \underline{w}(\eta, \alpha)$) will opt out, not buying an apartment or taking a loan; a moderately wealthy individual ($\underline{w}(\eta, \alpha) \leq w \leq \bar{w}(\eta, \alpha)$) will choose to pay down just the minimum; a sufficiently wealthy individual ($w > \bar{w}(\eta, \alpha)$) will choose to pay down strictly above the minimum, unconstrained by the minimum requirement.
4. **Policy Implication ii** The two cut-off levels satisfy $\frac{\partial \bar{w}_0}{\partial \eta} > 0$, $\frac{\partial \underline{w}_0}{\partial \eta} > 0$. When the minimum down payment is raised from η to $\eta' > \eta$, the cut-off wealth level that ensures opt-in $\underline{w}(\eta, \alpha)$ needs to be higher; the cut-off wealth level above which the choice of down payment is unconstrained by the minimum, $\bar{w}(\eta, \alpha)$, also needs to be higher.

The proofs are provided in Appendix I. The policy implications can be illustrated in Figure 2. Suppose

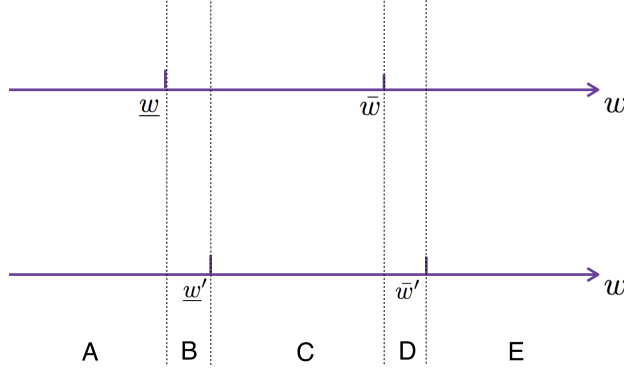


Figure 2: Raising the Minimum Down Payment

the minimum down payment is raised from η to η' . Before the raise, the cut-off wealth levels are \underline{w} and \bar{w} . After the raise, the two cut-off levels increase to \underline{w}' and \bar{w}' . The support of wealth is accordingly partitioned into five regions, A, B, C, D and E. Before the raise, borrowers whose wealths fall into region A will not opt in; borrowers whose wealths fall into regions B and C will opt in, paying down just the minimum; borrowers D and E will pay down strictly above the minimum. After the raise, borrowers B will now opt out; borrowers C will stay, having to pay more up front; borrowers D now become the new threshold borrowers; and borrowers E are not affected. It is clear that borrowers B, C and D are worse off under the higher minimum payment. B is worse off because they now quit the market. C and D are also worse off as they now have to pay down a larger amount (though monthly payments are smaller).⁹

In sum, at a raise, the poorest will stay out of the market; the richest will not be affected; the middle class, either were constrained and now constrained by higher minimum down payment requirement, or not constrained before but now forced to pay down a larger amount, get worse off. To quantify these effects, In section 5, we perform two counterfactual experiments: (1) raising the minimum to 60% for all pre-2011 second time buyers to examine how many quit the market and how much worse those who stay in the market get; and (2) removing the minimum requirement, or 0% down, to examine what default risks would have been if the amount of down payment (or the size of the loan) could be chosen freely.

3.0.1 Inferring Wealth Endowments

Given the observed down payment $down_i$ and characteristics α_i , we can draw inference on the unobserved endowment $w_{i,0}$. We only need that z_i^* monotonically increases on $w_{i,0}$ for any α_i at Θ . If this is true, then there exists an inverse function $w_{i,0} = z^{-1}(z_i^*, \alpha_i; \Theta)$.

Because (by definition) a non-threshold borrower's down payment is strictly above the required minimum, the observed down payment is the interior solution, $down_i = z_i^* = z(w_{i,0}, \alpha_i)$. Thus, a point inference is provided by

$$w_{i,0} = z^{-1}(z_i^*, \alpha_i; \Theta) = z^{-1}(down_i, \alpha_i; \Theta). \quad (6)$$

For a threshold borrower who paid down exactly $p_i\eta_i$, $z_i^* \leq p_i\eta_i$, we cannot recover a point solution for her

⁹Suppose that before the raise a borrower whose wealth falls into region C paid down $p\eta > z^*$, and $V^{in}(p\eta) < V^{in}(z^*)$. Because V^{in} is concave in z , $V^{in}(p\eta') < V^{in}(p\eta) < V^{in}(z^*)$. Similarly, suppose that before the raise a borrower whose wealth falls into region D paid down $z^* > p\eta$. After the raise she has to pay down $p\eta' > z^*$. Since z^* is the optimal down payment, $V^{in}(p\eta') < V^{in}(z^*)$.

initial wealth, as her z_i^* is not observed.¹⁰ Instead, the bounds of initial wealth can be inferred. The upper bound of her endowment is the wealth level that would result in a down payment just equal to the required minimum; intuitively, if she were wealthier, a payment strictly higher than the minimum requirement would have been observed; the lower bound is the wealth level that ensures opt-in at the observed minimum down payment, because if she were too poor, she would not have been able to afford the minimum requirement.

Formally, the upper bound of a threshold borrower i 's wealth endowment is

$$\bar{w}_{i,0} = z^{-1}(\eta_i p_i, \alpha_i; \Theta). \quad (7)$$

This is the highest possible wealth she could possess. Given the monotonicity of $z^{-1}(\cdot)$, it is clear that the actual initial wealth cannot be above this critical level, otherwise she would have paid strictly more than $p_i \eta_i$:

$$z_i^* \leq p_i \eta_i \Rightarrow w_{i,0} = z^{-1}(z_i^*, \cdot) \leq z^{-1}(p_i \eta_i, \cdot) = \bar{w}_{i,0}. \quad (8)$$

The lower bound of her endowment $\underline{w}_{i,0}$ ensures just opt-in:

$$\bar{V}_{i,1}^{in}(\underline{w}_{i,0} - p_i \eta_i, p_i(1 - \eta_i)R_i) = V_{i,0}^{out}(\underline{w}_{i,0}). \quad (9)$$

If endowment were lower than $\underline{w}_{i,0}$, the inside value at the required payment $p_i \eta_i$ will result in opt-out. Suppose the lower bound implied by this implicit function is

$$\underline{w}_{i,0} = w(\eta_i, \alpha_i). \quad (10)$$

The single-crossing condition ensures that there exists at most one such $\underline{w}_{i,0}$. Finally, for both threshold and non-threshold borrowers opt-in requires that the inside value at the observed down payment must be no less than the outside value. This condition is checked afterwards.

To summarize, for threshold borrowers, we compute the wealth bounds $[\underline{w}_{i,0}, \bar{w}_{i,0}]$ using the minimum requirement and the observed characteristics; for borrowers whose inside value function at the minimum requirement always lies above the outside value function for all $w_{i,0}$, we set $\underline{w}_{i,0} = 0$. For non-threshold borrowers, a point inference is provided by $z^{-1}(z_i^*, \alpha_i)$.

3.0.2 The Alternative Minimum Down Payment Requirements

Raising the minimum down payment

In one of the experiments we simulate outcomes when the required minimum down payment ratio is raised to a hypothetical level. It can be shown that, under a higher minimum ratio η_i , everything else equal the lowest wealth level that ensures opt-in is also higher:

$$\frac{\partial w(\eta_i, \alpha_i)}{\partial \eta_i} > 0. \quad (11)$$

The implication is that when the minimum ratio is raised from η_i to η'_i , there will be fewer borrowers taking loans; individuals whose endowments below $w(\eta'_i, \alpha_i)$ will choose the outside option. If a raise in

¹⁰The solution of the optimal down payment problem in (1) could be a corner solution, p_i , in which case the buyer takes no mortgages and is therefore unobserved in the mortgage data.

the minimum ratio causes a sizable effect in screening out presumably less wealthy borrowers, then the 2011 policy would be able to chill down the real estate market by excluding a large number of potential borrowers.

Zero-down required

In the other counterfactual experiment we simulate the outcome of threshold borrowers when the required minimum is zero percent (non-threshold borrower behavior will not change). Intuitively, under the zero-down policy, since a threshold borrower is not required to pay down as much, after down payment she will have at least as much wealth remaining as she does under the actual policy, and the counterfactual default rate should be at most as high. Actual and counterfactual variables are respectively denoted with superscript a and c , hereafter.

To simulate counterfactual default choices, we need to know the remaining wealth after down payment is made, which is the initial condition for the subsequent default choice making. For a threshold borrower, because her endowment is inferred to lie in an interval, the actual remaining wealth also lies in an interval, $w_{i,0}^{r,a} \in [\underline{w}_{i,0} - \eta_i p_i, \bar{w}_{i,0} - \eta_i p_i]$. Under the zero-down policy, the counterfactual and the actual upper bound of the remaining wealth will be the same, because at the highest possible endowment $\bar{w}_{i,0} = z^{-1}(\eta_i p_i, \alpha_i)$ the modeled optimal down payment is the same as the observed one, $z(z^{-1}(\eta_i p_i, \alpha_i), \alpha) = \eta_i p_i$.

In contrast, the counterfactual lower bound will be (weakly) higher than its actual counterpart. Under the zero-down policy, the counterfactual remaining wealth is $w_{i,0}^{r,c}(w_{i,0}) = w_{i,0} - z(w_{i,0}, \alpha_i)$, $w_{i,0} \in [\underline{w}_{i,0}, \bar{w}_{i,0}]$. This function monotonically increases on $w_{i,0}$, i.e., the rich individuals have more remaining wealth after down payment. It follows that the bounds of the counterfactual remaining wealth are

$$\begin{aligned}\bar{w}_{i,0}^{r,c} &= \bar{w}_{i,0} - z(\bar{w}_{i,0}, \alpha_i), \\ \underline{w}_{i,0}^{r,c} &= \underline{w}_{i,0} - z(\underline{w}_{i,0}, \alpha_i).\end{aligned}\tag{12}$$

Recall that the upper bound of endowment is the wealth level at which the minimum down payment is just the interior solution, $\eta_i p_i = z(\bar{w}_{i,0}, \alpha_i)$. Thus, compared to the actual remaining wealth, the upper bound of the counterfactual remaining wealth is the same:

$$\bar{w}_{i,0}^{r,c} = \bar{w}_{i,0} - z(\bar{w}_{i,0}, \alpha_i) = \bar{w}_{i,0} - \eta_i p_i = \bar{w}_{i,0}^{r,a}.\tag{13}$$

But since $\eta_i p_i = z(\bar{w}_{i,0}, \alpha_i) \geq z(\underline{w}_{i,0}, \alpha_i)$, the lower bound is weakly higher than the actual counterpart:

$$\underline{w}_{i,0}^{r,c} = \underline{w}_{i,0} - z(\underline{w}_{i,0}, \alpha_i) \geq \underline{w}_{i,0} - \eta_i p_i = \underline{w}_{i,0}^{r,a}.\tag{14}$$

4 Policy Evaluation: the 2011 Raise in Minimum Down Payment

4.1 Wealth Composition of the Buyers

For each subgroup, the *pdf* of endowed and remaining wealths, obtained by aggregating individual densities over all borrowers within the subgroup, are displayed in Figure 3. The *pdfs* of individual wealth endowment $h(w_{i,0})$ and remaining wealth $h(w_{i,0}^r)$ are constructed in the fashion described in Appendix VI. Threshold and non-threshold are marked by blue and red, respectively. The distributions show that non-threshold borrowers are in general endowed with more wealth and, after down payment, have more wealth

remaining than the threshold borrowers. Across all six subgroups, regardless of borrower type, the densities of non-threshold borrowers' endowment and remaining wealth both lie to the right of those of threshold borrowers; the averages wealth levels of the former are also higher than the latter (Table 4). Another finding confirms that the second time buyers are indeed wealthier than first time buyers: the endowment density curves of the second time buyers lie to the right of the first time buyers' (Figures 3a, 3c, 3e), and the average endowments of second time buyers, 602.80k and 905.36k for pre- and post-2011, respectively, are also higher than first time's 538.13k. However, the pre-2011 second time buyers do not appear significantly wealthier than first time buyers. If one is willing to believe that first time borrowers who purchase apartments mainly for dwelling purposes are representative urban middle class, then the 2011 policy of raising the minimum down payment seems to be reasonably grounded: if a good number of merely moderately wealthy individuals (i.e., the pre-2011 second time buyers) are purchasing their second homes, the market is probably overheated. In contrast, the post-2011 second time borrowers seem to be truly wealthy, with an average wealth endowment of 905k. This could support the theory that property purchases of the second time buyers may not be for dwelling but investment or arbitrage.

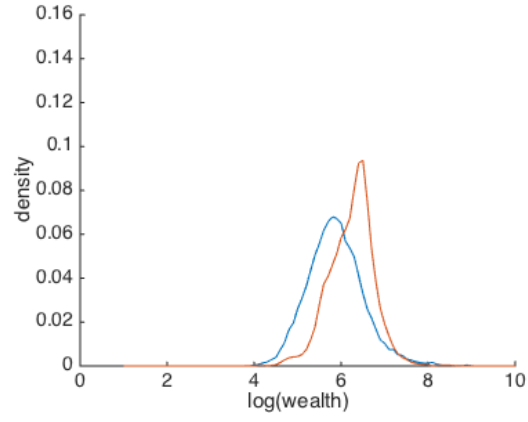
4.2 The Raise in Minimum Down Payment to 60%

The raise in the minimum ratio has different impacts on the second time borrowers with different endowment levels. The model in section 3 implies that after the raise, low-wealth individuals will opt-out; the "middle class" with moderate wealth will still opt in, paying exactly 60% of the property price; the "rich" who would have paid strictly above 60% anyway before the raise are not affected by the raise.

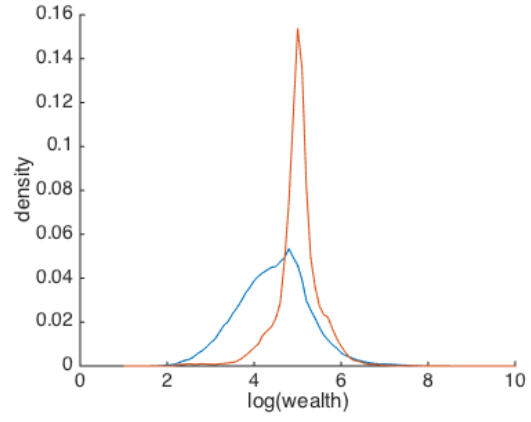
Comparing Figures 3e and 3c, it is found that the density curve of endowment has shifted rightwards after 2011, implying that the post-2011 second time buyers are generally wealthier than the pre-2011 second time buyers. The 2011 raise in minimum ratio managed to screen out low-wealth borrowers. After 2011, there are much fewer borrowers with endowment below $\exp(6)$, whereas before 2011 half of the borrowers have endowments below $\exp(6)$.

However, the screen-out does not prevent the emergence of high-wealth borrowers. The rightward shift of the density is caused by not only the withdrawing of low-wealth borrowers, but also the incoming of the rich. Figure 2c shows that after 2011 there emerged a large number of non-threshold borrowers with down payments strictly greater than 60%. Indeed, the choice of a large amount of down payment may be a result of the higher interest rate after 2011. However, the inferred endowment density curves in Figure 3e imply that about half of the post-2011 second time non-threshold borrowers have endowment between $\exp(7)$ and $\exp(8)$, whereas in Figure 4c, only a small portion of the pre-2011 have such big wealth. This unprecedented proportion of riches on the right end of the density curve parallels the large number of non-threshold borrowers observed after 2011 (Figure 1a). The emergence of the new rich may have to do with China's recent urbanization with rural rich settling down in metropolitan areas.

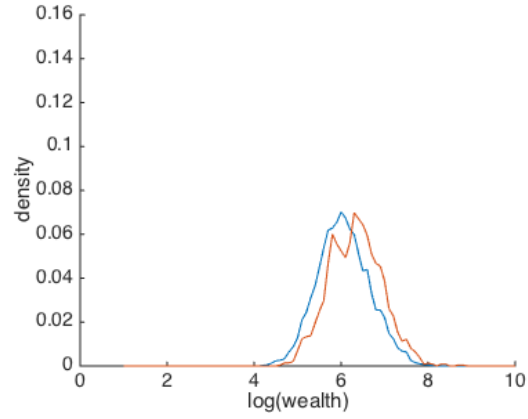
The "middle class" refers to the post-2011 second time threshold borrowers, marked by the red curve in Figure 3e, with endowment roughly ranging from $\exp(6)$ to $\exp(8)$. Under the 60% minimum requirement, if a borrower's endowment is not too high (so that she will not pay down strictly above 60%), and not too low (so that she will not opt out), she pays down just the minimum. Had the minimum requirement been lower, she could have paid some amount below 60%, and would have had more wealth remaining after down payment. With very little wealth remaining, she will struggle with lower consumption and a high risk of default.



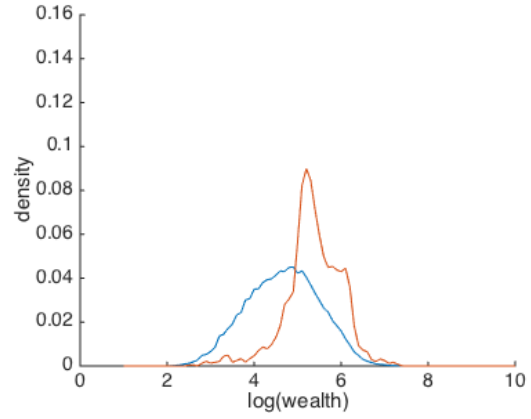
(a) first time buyers, endowment



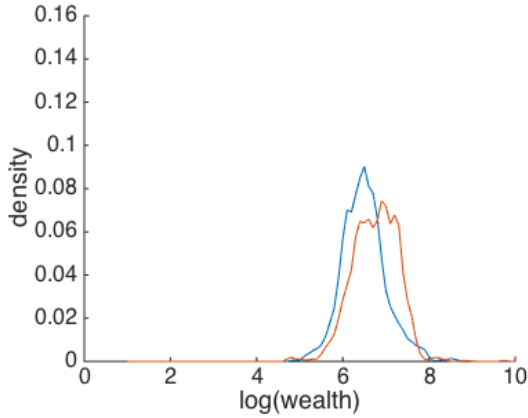
(b) first time buyers, after down payment



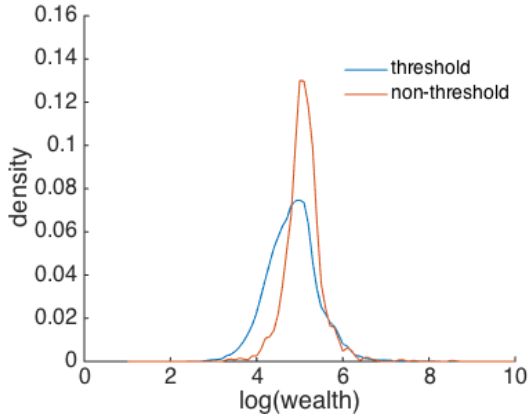
(c) second time buyers, pre-2011, endowment



(d) second time buyers, pre-2011, after down payment



(e) second time buyers, post-2011, endowment



(f) second time, post-2011, after down payment

Figure 3: Distributions of Inferred Endowments and Remaining Wealth

5 Counterfactual Experiments

5.1 Eliminating the Minimum Down Payment Requirement

The threshold borrowers in general have higher probabilities of default than the non-threshold borrowers. The model in Section 3 suggests that the minimum down payment requirement takes too much liquid wealth away, leaving them too little wealth to deal with future uncertainties. However, to show that it is the minimum down payment requirement that makes them pay down too much and therefore suffer from high default probabilities, one needs a counterfactual experiment, in which the observed threshold borrowers' wealth levels and borrower characteristics are controlled for. A straightforward experiment is what the threshold borrowers' default probabilities would have been, if there were no such minimum down payment requirement (the non-threshold borrowers' behaviors would of course not change if we eliminate the minimum). This experiment is run on the first time 10-, 20- and 30-year threshold borrowers. We compute each observed threshold borrower's choice of down payment when she can freely choose z_i^* , given the inferred bounds of wealth endowment. Presumably, this freely-chosen down payment should be smaller than the actual one. Thus, if the remaining wealth is higher under the zero-down policy, the default probabilities are expected to be lower.

For a threshold borrower, since her wealth is inferred to lie within an interval, the choice of down payment under the zero-down policy also lies in an interval, with the highest possible one equal to the observed threshold. Thus, the counterfactual and the actual remaining wealth have the same upper bound, but the counterfactual lower bound will be (weakly) greater than its actual counterpart.

In practice, for each observed threshold borrower, first we compute the counterfactual optimal down payment and the per-period payment at the two bounds of her endowment, then compute the counterfactual bounds of the remaining wealth, and finally simulate the probabilities of default with the initial condition that the remaining wealth is uniformly distributed between these bounds. The result is presented in Figure 4 and the upper half of Table 5. Figure 4a shows the aggregate remaining wealth distributions under the actual and the zero-down policies; Figures 4b, 4c and 4d show the survival functions of the 10-, 20- and 30-year groups, contrasting with the actual survival functions (yellow curves).

The counterfactual outcome is summarized in the upper half of Table 5. For the 5,540 observed threshold borrowers, the counterfactual average remaining wealth is 225.84k CNY; that is 82.32% higher than in the actual case. With more wealth remaining at the beginning of the mortgage, the borrowers are less vulnerable to future uncertainties. The predicted default rates under the zero-down policy for the 10-, 20- and 30-year groups are 0.0308, 0.0401 and 0.0375, which are 7.68%, 14.86% and 22.31% lower than in the actual case, respectively. It is worth noticing that this counterfactual experiment, which is lowering the minimum down payment ratio, is able to predict the counterfactual outcomes for only the observed individuals, but not the censored individuals. In the next counterfactual experiment, we simulate the outcomes when raising the minimum down payment ratio, thus encountering no such censoring issue.

5.2 Raising the Minimum Down Payment Ratio

The effect of the 2011 minimum down payment raise is two-fold: on the one hand, the monthly payment will be smaller as a bigger portion of the property price is paid up front; on the other hand, the wealth of the borrower is rapidly reduced by the down payment, leaving her vulnerable to future shocks. Though the policy of raising the minimum down payment may have managed to filter out the low-wealth borrowers and

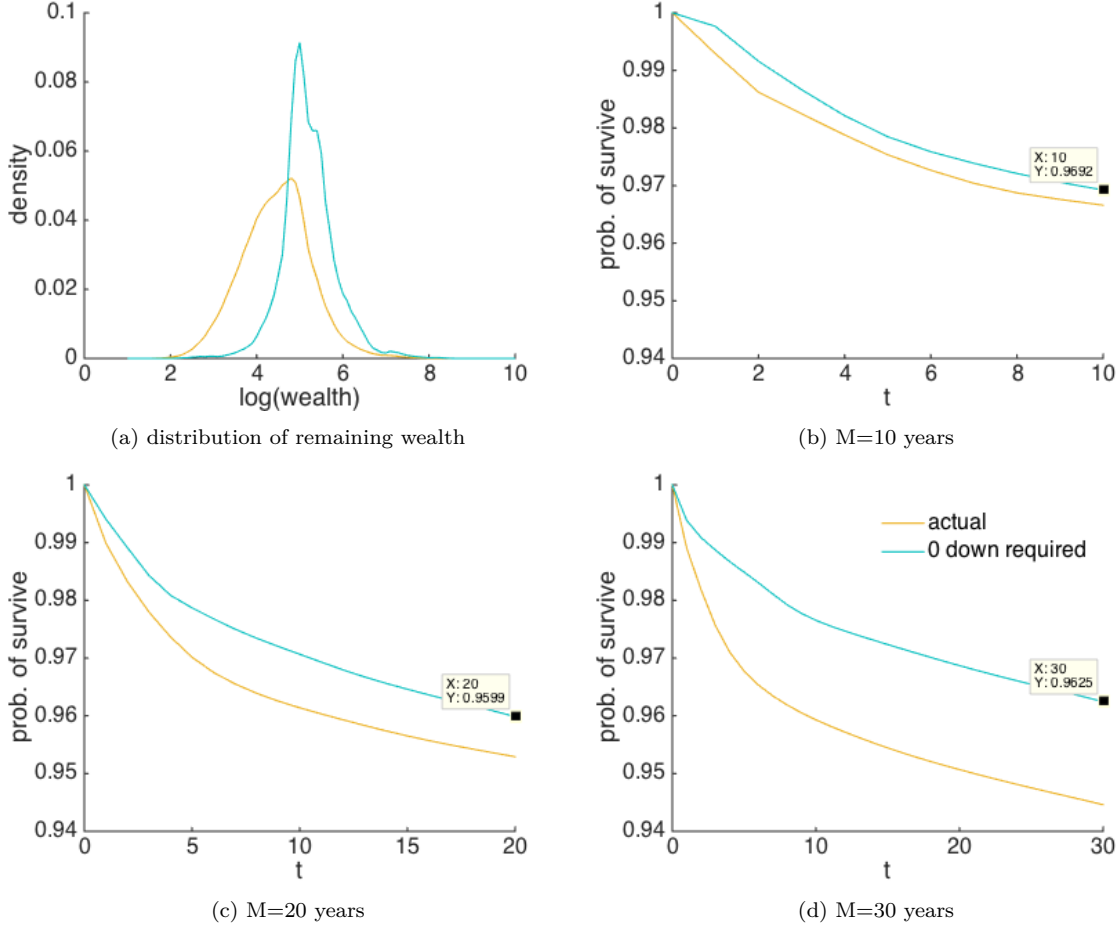


Figure 4: Counterfactual 1, 0% Down Is Required

thus to some extent reduce potential real estate bubbles, this second counterfactual experiment highlights the negative impacts: more defaults and bank losses.

To examine the policy impact on second time buyers, one cannot simply compare the predicted default probabilities of the observed pre- and post-2011 second time borrowers, because of censoring. To see the pure policy impact on default probabilities, one should simulate the outcomes while controlling for borrower heterogeneities, observed and unobserved.

Specifically, we raise the minimum ratio to 60%, which resembles the 2011 policy,¹¹ for all pre-2011 second time buyers and simulate their default rates (regardless of the minimum they were actually eligible for). The observed borrower characteristics α_i and the inferred wealth endowment (point or bounds) are taken as given, except that the per-period down payment is conditional on the new down payment choice¹². Because all historical minimum ratios required for pre-2011 second time are strictly lower (20% to 50%), the down payment choice and default choice probabilities of those who actually pay down strictly above 60% will

¹¹The 2011 policy also requires individual loan interest rates to be at least 10% above the base rate, $dev_i \geq 0.1$.

¹²Taking the supply-side variables as given may cause endogeneity issue. For example, at the higher minimum ratio a borrower who chose an expensive apartment would now switch to a less pricey one; and property prices would in turn fall. Modeling the apartment choice and supply-side choices (property sellers and the bank) would be an interesting extension of the partial-equilibrium model in this paper.

Table 2: Counterfactual Experiments: 0% down and 60% down

	Actual	Counterfactual
<i>Counterfactual 1: 0% down required</i>		
# of Borrowers	5540	5540
Average remaining wealth	123.87	225.84
Default Prob.		
M=10	3.34%	3.08%
M=20	4.71%	4.01%
M=30	5.54%	3.75%
<i>Counterfactual 2: 60% down required</i>		
# of Borrowers		
opt-in, observed	2,250	-
opt-in, simulated	2,197	1,123
threshold	965	840
non-threshold/unaffected	1,232	283
quitted	-	1,074
Average endowment	602.80k	711.29k
Average remaining wealth	210.08k	198.42k
Average default prob.	3.29%	38.13%

not change. For each of the remaining borrowers, we simply determine at the 60% down payment whether or not she will opt in; if yes, then subtract the counterfactual down payment from endowment and simulate the default probabilities.

The model in section 3 suggests that, at a higher minimum down payment ratio, the lower bound of wealth endowment, where the borrower is indifferent between opting in and out, is also higher (see eq. (11)). It follows that the least-wealthy individuals will opt out under the higher minimum ratio. A raise in the minimum ratio will have different impacts on individuals of different endowment levels: it will have no impact on the borrowers who chose just or strictly above the new minimum (the unaffected); for those who chose below this level, they will either opt out, if the new minimum is unaffordable (the quitted), or stay and choose exactly that new minimum (the new threshold). The new threshold borrowers will have less remaining wealth after down payment, and therefore be more likely to default compared to the actual case. Figure 5 depicts the simulated *pdfs* of endowments and remaining wealth levels under the actual and the counterfactual 60% minimum ratios. The lower half of Table 5 summarizes the counterfactual outcome.

Since a higher minimum down payment ratio will screen out the low-wealth individuals, the density curve of endowments of the opt-in individuals is expected to shift rightwards. The simulated wealth *pdf* in Figure 11a confirms this prediction. It also has a smaller deviation. The average endowments under the actual and 60% minimum ratios are respectively 602.80k and 711.29k CNY. Only 1,123 borrowers (out of 2,197) stay in market at the 60% minimum ratio.¹³ Thus the 2011 policy would exclude (less wealthy) potential real

¹³As discussed in section 3, the condition that ensures opt-in ($E[V^{in}] \geq V^{out}$) is checked afterwards. Using the parameter

estate buyers. On the other hand, there would be 840 “new” threshold borrowers who now choose to pay down exactly 60%. The number of unaffected individuals is 283.

Even after the least-wealthy individuals have been screened out by the 60% minimum ratio, the average remaining wealth of the staying individuals is 198.42k, lower than the actual 210.08k (Figure 11b). As noted in section 3, lower wealth at the beginning of the mortgage indicates higher probabilities of default at all t during the course of loan repayment. The default rate of the staying 1,123 borrowers (the 840 new threshold plus the 283 unaffected) is predicted to be as high as 38.13%. This is mainly due to the much higher default probabilities of the new threshold borrowers (recall that the 283 unaffected borrowers’ remaining wealth levels and default probabilities remain the same). Furthermore, not only are the probabilities of full repayment lower, but default would also occur sooner; this means a bigger loss for the lending bank.

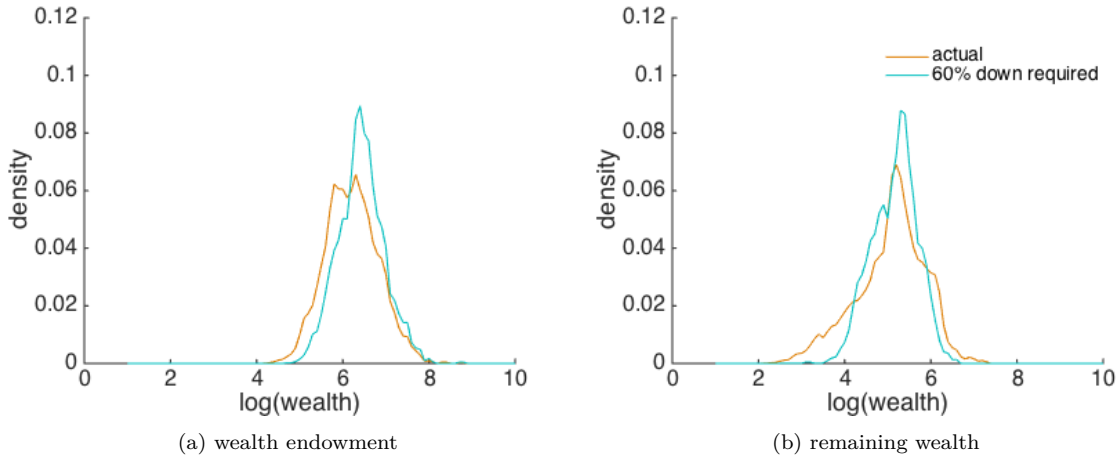


Figure 5: Counterfactual 2, Raising the Minimum to 60%

6 Conclusion

In general, mortgage down payment (deposit) serves as an insurance for the lending institute: it makes it more likely to recover the balance in the event of default. However, in China the required down payment by the centralized policy creates a huge burden for Chinese households; homes are expensive, with the price-to-income ratio 25 to 30 in major Chinese cities, and the minimum down payment ratio ranges from 20% to 60%.

The aims of the Chinese housing mortgage policy are believed to be macroeconomic – to maintain the real estate market prices (which benefits state banks), to boost prosperity of the construction industry (one of the major driving forces of GDP, employment and urbanization), to diminish real estate bubbles, etc. – whereas the well-being of individual borrowers seem to be of less concern. Among other mortgage contract variables, the minimum down payment is frequently adjusted in the recent decade, in order to either boost real estate demand or to cool down the market.

Our model shows a severe adverse selection problem in the current policy: under the uniform contract, (conditional on property value) those who pay down just the minimum are more likely to default than those

estimates, there are 2,197 out of 2,250 observed pre-2011 second time borrowers are predicted to opt in under the actual policy.

who made larger amounts. We also (partially) recovered liquid wealth levels in order to further analyze the centralized policy.

One implication of the model is that, regardless of the size of the minimum down payment requirement, it screens out the least-wealthy borrowers, and at the same time takes too much wealth away from the moderately wealthy individuals. The results suggest that the efficacy of the 2011 raise in the minimum down payment, aiming to cool down the market, is doubtful, because the uniform down payment requirement can do nothing to prevent the rich from entering the market. Moreover, the policy maker may have overlooked or underestimated the high long-run default risks of the individuals whose wealths are drained by the 60% minimum. The policy maker thus faces a trade-off between the current real estate market stability and long-run risks.

Another implication is that any raise in the minimum will make some non-threshold borrowers to become threshold borrowers. As shown in the zero-down experiment, if there were no minimum requirement, the borrowers would have significantly higher wealth remaining and default rates would be substantially lower. Certainly, the purpose of the paper is not to recommend completely removing the down payment requirement. Instead, it highlights the struggles of Chinese households when facing both high home prices and high down payment requirements.

In reality, the choice of apartment (price and housing characteristics) could be sensitive to mortgage policy. For instance, under high interest and high minimum down payment, borrowers could downgrade the apartment she has in mind, in order to preserve more liquid wealth to deal with future uncertainties. If the effect of apartment re-optimization is large, then the simulations could considerably overestimate the default rates of the staying borrowers under a tougher policy. The choice of apartment would be an interesting extension, and could be modeled in a stage prior to the choice of down payment, taking bank-chosen variables as given. This paper builds a partial equilibrium model, assuming all supply-side variables, including property prices and mortgage contract variables are fixed. A full equilibrium model consisting of both supply and demand sides and policy maker would be of future research interest.

References

- P. Arcidiacono and J. B. Jones. Finite mixture distributions, sequential likelihood and the em algorithm. *Econometrica*, 71(3):933–946, 2003.
- P. Arcidiacono and R. A. Miller. Ccp estimation of dynamic discrete choice models with unobserved heterogeneity. *Tepper School of Business*, page 319, 2008.
- O. P. Attanasio, P. Koujianou Goldberg, and E. Kyriazidou. Credit constraints in the market for consumer durables: Evidence from micro data on car loans. *International Economic Review*, 49(2):401–436, 2008.
- L. M. Ausubel. The failure of competition in the credit card market. *The American Economic Review*, pages 50–81, 1991.
- L. M. Ausubel. Adverse selection in the credit card market. Technical report, working paper, University of Maryland, 1999.
- P. Bajari, C. S. Chu, D. Nekipelov, and M. Park. A dynamic model of subprime mortgage default: estimation and policy implications. Technical report, National Bureau of Economic Research, 2013.

- D. Brito, P. Pereira, and T. Ribeiro. Estimating the demand for mortgage loans with an application to merger analysis. Technical report, Working Paper, 2009.
- J. Y. Campbell and J. F. Cocco. Household risk management and optimal mortgage choice. *Quarterly Journal of Economics*, 118(4), 2003.
- J. Cao, B. Huang, and R. N. Lai. On the effectiveness of housing purchase restriction policy in china: a difference in difference approach. *Available at SSRN 2584275*, 2015.
- A. DEATON. Saving and liquidity constraints. *Econometrica*, 59(5):1221–1248, 1991.
- L. Deng, Q. Shen, and L. Wang. Housing policy and finance in china: A literature review. *US Department of Housing and Urban Development*, 2009.
- Y. Deng. Mortgage termination: An empirical hazard model with a stochastic term structure. *The Journal of Real Estate Finance and Economics*, 14(3):309–331, 1997.
- Y. Deng and P. Liu. Mortgage prepayment and default behavior with embedded forward contract risks in china’s housing market. *The Journal of Real Estate Finance and Economics*, 38(3):214–240, 2009.
- Y. Deng, J. M. Quigley, and R. Order. Mortgage terminations, heterogeneity and the exercise of mortgage options. *Econometrica*, 68(2):275–307, 2000.
- Y. Deng, D. Zheng, and C. Ling. An early assessment of residential mortgage performance in china. *The Journal of Real Estate Finance and Economics*, 31(2):117–136, 2005.
- W. M. Edelberg. Risk-based pricing of interest rates in household loan markets. *SSRN Working Paper Series*, 2004a.
- W. M. Edelberg. Testing for adverse selection and moral hazard in consumer loan markets. *SSRN Working Paper Series*, 2004b.
- L. Einav, M. Jenkins, and J. Levin. Contract pricing in consumer credit markets. *Econometrica*, 80(4): 1387–1432, 2012.
- L. Einav, A. Finkelstein, S. P. Ryan, P. Schrimpf, and M. R. Cullen. Selection on moral hazard in health insurance. *The American economic review*, 103(1):178–219, 2013.
- C. Foster and R. Vanorder. An option-based model of mortgage default. *Housing Finance Review*, 3(4): 351–372, 1984.
- R. J. Gary-Bobo and S. Larribeau. A structural econometric model of price discrimination in the french mortgage lending industry. *International Journal of Industrial Organization*, 22(1):101–134, 2004.
- D. B. Gross and N. S. Souleles. Do liquidity constraints and interest rates matter for consumer behavior? evidence from credit card data. *Quarterly Journal of Economics*, pages 149–185, 2002.
- J. Heckman and B. Singer. A method for minimizing the impact of distributional assumptions in econometric models for duration data. *Econometrica: Journal of the Econometric Society*, pages 271–320, 1984.
- J. J. Heckman. The incidental parameters problem and the problem of initial conditions in estimating a discrete time-discrete data stochastic process. 1981.

- I. Hendel and A. Nevo. Measuring the implications of sales and consumer inventory behavior. *Econometrica*, 74(6):1637–1673, 2006.
- D. S. Johnson, J. A. Parker, and N. S. Souleles. Household expenditure and the income tax rebates of 2001. *The American Economic Review*, 96(5):1589–1610, 2006.
- F. T. Juster and R. P. Shay. Consumer sensitivity to finance rates. In *Consumer Sensitivity to Finance Rates: An Empirical and Analytical Investigation*, pages 6–46. NBER, 1964.
- D. Karlan and J. Zinman. Observing unobservables: Identifying information asymmetries with a consumer credit field experiment. *Econometrica*, 77(6):1993–2008, 2009.
- M. P. Keane and K. I. Wolpin. The career decisions of young men. *Journal of political Economy*, 105(3):473–522, 1997.
- G. Li. *China Household Finance Survey*. China Household Finance Center, Southwestern University of Finance and Economics, China, 2011.
- A. Pakes. Patents as options: Some estimates of the value of holding european patent stocks. *Econometrica: Journal of the Econometric Society*, pages 755–784, 1986.
- E. S. Schwartz and W. N. Torous. Prepayment and the valuation of mortgage-backed securities. *The Journal of Finance*, 44(2):375–392, 1989.
- N. S. Souleles. The response of household consumption to income tax refunds. *The American Economic Review*, 89(4):947–958, 1999.
- G. Ye, G. Deng, and Z. Li. Mortgage rate and choice of mortgage length: A quasi-experimental evidence from chinese transaction-level data. *Journal of Housing Economics*, 25:96–103, 2014.
- S. P. Zeldes. Consumption and liquidity constraints: an empirical investigation. *The Journal of Political Economy*, pages 305–346, 1989.

Appendix I

Assumption 1. (A1)

$$\frac{d\bar{V}^{in}(w_0)}{dw_0} > \frac{dV^{out}(w_0)}{dw_0}, \forall w_0,$$

i.e., if \bar{V}^{in} and V^{out} cross, they cross at most once.

Proposition 1.

$$0 < \frac{dz^*(w_0)}{dw_0} < 1.$$

Taking partial derivative w.r.t. w_0 ,

$$\frac{dz(w_0)}{dw_0} = \frac{\frac{\partial^2 \bar{V}^{in}}{\partial w_0^2} + R \frac{\partial^2 \bar{V}^{in}}{\partial x \partial w}}{\frac{\partial^2 \bar{V}^{in}}{\partial w_0^2} + 2R \frac{\partial^2 \bar{V}^{in}}{\partial x \partial w_0} + R^2 \frac{\partial^2 \bar{V}^{in}}{\partial x^2}} \quad (15)$$

The second-order condition for z^* ensures the denominator is negative. The sign of the numerator is unknown. The two inequalities are checked numerically, by computing z^* at a range of w_0 for various α .

Proposition 2. *Given minimum down payment requirement $0 < \eta < 1$ and borrower characteristics α , there exists two cut-off levels of wealth, $\underline{w}_0(\eta, \alpha)$ and $\bar{w}_0(\eta, \alpha)$ satisfying $\underline{w}_0(\eta, \alpha) \leq \bar{w}_0(\eta, \alpha)$, such that*

1. for $0 < w < \underline{w}_0(\eta, \alpha)$, opt out;
2. for $\underline{w}_0(\eta, \alpha) \leq w \leq \bar{w}_0(\eta, \alpha)$, $z^*(w) < p\eta$ and down = $p\eta$;
3. for $w > \bar{w}_0(\eta, \alpha)$, $z^*(w) > p\eta$.

Proof.

Step 1. \underline{w} satisfies $\bar{V}^{in}(\underline{w} - p\eta, R(p - p\eta)) = V^{out}(\underline{w})$. Also notice that

$$z^*(w) < z^*(\underline{w}) \leq z^*(\bar{w}) = p\eta.$$

By envelope theorem $\underline{V}_1^{in}(\underline{w} - z^*(\underline{w}), R(p - z^*(\underline{w}))) = \frac{d\bar{V}^{in}(\underline{w})}{d\underline{w}}$. Since $z^*(\underline{w}) < p\eta$, concavity of $\bar{V}^{in}(z)$ implies

$$\bar{V}_1^{in}(\underline{w} - p\eta, R(p - p\eta)) > \frac{\bar{V}^{in}(\underline{w})}{d\underline{w}} > \frac{dV^{out}}{d\underline{w}}$$

The last inequality comes from A1. Thus, for $w < \underline{w}$

$$\bar{V}^{in}(w - p\eta, R(p - p\eta)) - V^{out}(w) < \bar{V}^{in}(\underline{w} - p\eta, R(p - p\eta)) - V^{out}(\underline{w}) = 0,$$

i.e., any individual with $w < \underline{w}$ will not opt in.

Step 2. \bar{w} satisfies $z^*(\bar{w}) = p\eta$, and $\bar{V}^{in}(\bar{w}) \geq V^{out}(\bar{w})$. Then for any $w > \bar{w}$,

$$\bar{V}^{in}(w) > V^{out}(w),$$

and

$$z^*(w) > z^*(\bar{w}) = p\eta.$$

i.e., any individual with $w > \bar{w}$ will opt in and pay down more.

Step 3. Given steps 1 and 2, it must be true that $\underline{w} \leq \bar{w}$. Suppose that $\underline{w} = \bar{w}$. Then by construction

$$\bar{V}^{in}(\underline{w} - p\eta, R(p - p\eta)) = V^{out}(\underline{w}) = \bar{V}^{in}(\bar{w}) \geq V^{out}(\bar{w}).$$

Because $V^{out}(w)$ monotonically increases on w ,

$$\bar{V}^{in}(\bar{w}) = V^{out}(\bar{w}),$$

i.e., there exists a unique w where the choice of down payment is exactly $p\eta$.

Now suppose that $\underline{w} < \bar{w}$. Suppose $\underline{w} < w < \bar{w}$. Since

$$z^*(w) < z^*(\bar{w}) = p\eta,$$

the individual with w will choose $p\eta$ iff.

$$\bar{V}^{in}(w - p\eta, R(p - p\eta)) > V^{out}(\underline{w}).$$

The above inequality holds, because

$$\bar{V}^{in}(w - p\eta, R(p - p\eta)) > \bar{V}^{in}(\underline{w} - p\eta, R(p - p\eta)).$$

Proposition 3.

$$\frac{\partial \bar{w}_0}{\partial \eta} > 0, \frac{\partial \underline{w}_0}{\partial \eta} > 0$$

Proof.

The lowest possible wealth that ensures opt-in is given by

$$\bar{V}^{in}(\underline{w}_0 - p\eta, (p - p\eta)R) = V^{out}(\underline{w}_0),$$

Let f_1 and f_2 denote the partial derivative of function $f(\cdot, \cdot)$ with respect to the 1st and 2nd argument, respectively. We have,

$$\frac{d\underline{w}_0}{d\eta} = p_i \frac{-\bar{V}_1^{in} - R\bar{V}_2^{in}}{V_1^{out} - \bar{V}_1^{in}}. \quad (16)$$

By the envelope theorem $\bar{V}_1^{in}(w(z^*), x(z^*)) = d\bar{V}^{in}/dw_0$. And since $p\eta > z^*(w)$,

$$\bar{V}_1^{in}(w - p\eta, x(p\eta)) > \bar{V}_1^{in}(w - z^*(w), x(z^*(w))) = V_1^{out}(w).$$

Thus the denominator in eq. 16 is negative. Next, notice that at the optimal down payment $z^*(\bar{w}_0)$,

$$-\bar{V}_1^{in}(w_1(z^*), x_i(z^*)) - R\bar{V}_2^{in}(w_1(z^*), x(z^*)) = 0,$$

Since $p\eta > z^*$ and \bar{V}^{in} is strictly concave,

$$-\bar{V}_1^{in}(w_1(p\eta), x(p\eta)) - R\bar{V}_2^{in}(w_1(p\eta), x(p\eta)) < 0.$$

Both the numerator and the denominator in (16) are negative.

Next, since at \bar{w} ,

$$z^*(\bar{w}) = p\eta.$$

Taking partial derivative w.r.t. η ,

$$\frac{\partial z^*(\bar{w}(\eta))}{\partial \bar{w}} \frac{\partial \bar{w}(\eta)}{\partial \eta} = p.$$

Thus

$$\frac{\partial \bar{w}(\eta)}{\partial \eta} > 0.$$

Proposition 4. *Suppose that A1 is satisfied, then*

$$\frac{\partial \sigma_{it}}{\partial w_{it}} > 0.$$

Proof.

The choice probability of default is

$$\sigma_{it}(t, w_{it}, \alpha_i) = \frac{\exp(V^{in}(t, w_{it}, h_i, \alpha_i))}{\exp(V^{in}(t, w_{it}, h_i, \alpha_i)) + \exp(V^{out}(h_i^*, w_{it}, \alpha_i))}.$$

Thus,

$$\frac{\partial \sigma_{it}}{\partial w_{it}} = \sigma_{it}(1 - \sigma_{it}) \left(\frac{\partial \bar{V}_{it}^{in}}{\partial w_{it}} - \frac{\partial V_{it}^{out}}{\partial w_{it}} \right) > 0.$$